Efficient schemes for reducing imperfect collective decoherences

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We propose schemes that are efficient when each pair of qubits undergoes some imperfect collective decoherence with different baths. In the proposed scheme, each pair of qubits is first encoded in a decoherence-free subspace composed of two qubits. Leakage out of the encoding space generated by the imperfection is reduced by the quantum Zeno effect. Phase errors in the encoded bits generated by the imperfection are reduced by concatenation of the decoherence-free subspace with either a three-qubit quantum error correcting code that corrects only phase errors or a two-qubit quantum error detecting code that detects only phase errors, connected with the quantum Zeno effect again.

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Information processing with quantum bits (qubits), e.g., quantum computing, quantum cryptography, and quantum gambling is, a novel technique that solves some classically intractable problems [1]- [6]. However, in order to make quantum information processing involving many qubits practical, some methods for reducing decoherence (MRDs) are indispensable. Among these, there are quantum error correcting codes (QECCs) [7]- [14], decoherence-free subspaces (DFSs) [15]- [20], the quantum Zeno effect (QZE) [22,23], [1] and dynamical suppression of decoherence [27].

If the provisos for DFSs are fulfilled, DFSs are more efficient than QECCs or the QZE in respect to the amount of other necessary resources as well as the number of qubits. The robustness of DFSs against perturbation of replica symmetry is shown in Refs. [19,20]. Indeed, qubits in collective decoherence with imperfect replica symmetry can be preserved with concatenation of DFSs with QECCs [21]. However, the efficiencies of various MRD's depend on the decoherence model. So devising an optimal scheme that appropriately combines existing MRDs for a given decoherence model will be important in the design of quantum information processors. In this paper, we propose a scheme that is efficient when each pair of qubits undergoes imperfect collective decoherence with different baths (cluster decoherence [20,21]). We start with a subspace composed of two qubits which is decoherence-free against a certain interaction that generates only phase errors. Other interactions assumed small but non-negligible make the encoded states leak out of the DFS, and the interactions generate phase errors in the encoded qubits. The leakage is reduced by the QZE. The phase errors in the encoded qubits are corrected by concatenating the DFS with a three-qubit QECC that corrects only phase errors [11] or by concatenating the DFS with a two-qubit quantum error detecting code that detects only phase errors and by QZE again.

The dynamics of the qubits and bath is governed by

$$\mathbf{H}_T = \mathbf{H}_S + \mathbf{H}_B + \mathbf{H}_I, \tag{0.1}$$

where $\mathbf{H}_T$, $\mathbf{H}_S$, and $\mathbf{H}_B$ denote the total, the system, and the bath (or environment) Hamiltonian, respectively, and $\mathbf{H}_I$ is the interaction Hamiltonian. First, we consider the following simple model.

$$\begin{aligned} \mathbf{H}_S &= \epsilon(\sigma_1^z + \sigma_2^z), \\ \mathbf{H}_I &= \lambda(\sigma_1^z + \sigma_2^z) \otimes V_z, \end{aligned} \tag{0.2}$$

where σ_i^z $(i = 1, 2)$ are Pauli spin operators, V_z is the bath operator coupled to the degree of freedom, and $\mathbf{H}_B$ is arbitrary. The type of Hamiltonian in Eq. (0.2), which corresponds to a special case of the spin-boson problem, has been used by many authors to model decoherence despite its simplicity [27,28,16]: This model describes a decohering mechanism with only phase errors. Amplitude errors would involve a time scale much longer than that of phase errors in some real physical systems [16]. (Later, we will treat more general models.) We can easily see that a subspace spanned by $|01\rangle$ and $|10\rangle$ satisfies the DFS condition in the case of the interaction given by Eq. (0.2): $(\sigma_1^z + \sigma_2^z)|01\rangle = 0|01\rangle$ and $(\sigma_1^z + \sigma_2^z)|10\rangle = 0|10\rangle$. [2] Therefore,

$$\begin{aligned} &\mathbf{H}_T[(\alpha|01\rangle + \beta|10\rangle) \otimes |\Psi_b(0)\rangle] \\ &\quad = (\alpha|01\rangle + \beta|10\rangle) \otimes [0(\epsilon + \lambda V_z) + \mathbf{H}_B]|\Psi_b(0)\rangle, \end{aligned} \tag{0.3}$$

and as a result

$$\begin{aligned} &\exp[-i\mathbf{H}_T T_0](\alpha|01\rangle + \beta|10\rangle) \otimes |\Psi_b(0)\rangle \\ &\quad = (\alpha|01\rangle + \beta|10\rangle) \otimes \exp[-i\mathbf{H}_B T_0]|\Psi_b(0)\rangle. \end{aligned} \tag{0.4}$$

[1] The QZE was discovered by Misra and Sudersan [24]. The use of the QZE for combating decoherence was first suggested by Zurek [25], and it is a part of a scheme considered by Barenco et al. [26].

[2] It is noted that another condition should be additionally satisfied in order that some subspaces become decoherence-free: the system Hamiltonian $\mathbf{H}_S$ does not make qubits leak out of the subspace. Otherwise, $\mathbf{H}_S$ needs to be eliminated to satisfy this condition by the method proposed in Ref. [17].

Here we can see that the qubits indeed do not decohere. So Span$[|01\rangle, |10\rangle]$ (the subspace that $|01\rangle$ and $|10\rangle$ span) can be used to encode one qubit. That is, we can encode a qubit $\alpha|0\rangle + \beta|1\rangle$ into, for example, the state

$$|\Psi_{enc}\rangle = \alpha|01\rangle + \beta|10\rangle. \tag{0.5}$$

It is clear that this subspace is sufficient for preventing decoherence provided that the system and bath are perfectly governed by Eq. (0.2). However, in real systems there are some small perturbative interactions that are not included in Eq. (0.2). When the perturbative interaction is non-negligible, its effect must be reduced by some method that we will describe. Now, let us consider a more general decoherence model:

$$\begin{aligned} \mathbf{H}_S &= \epsilon_1\sigma_1^z + \epsilon_2\sigma_2^z, \\ \mathbf{H}_I &= [(\lambda_1^z\sigma_1^z + \lambda_2^z\sigma_2^z) \otimes V_z + (\lambda_1^+\sigma_1^+ + \lambda_2^+\sigma_2^+) \otimes V_+ + \\ &\quad (\lambda_1^-\sigma_1^- + \lambda_2^-\sigma_2^-) \otimes V_-]. \end{aligned} \tag{0.6}$$

Here, σ_i^j $(j = z, +, -)$ are Pauli spin operators and V_j are the bath operators coupled to these degrees of freedom. We assume that $\Delta\epsilon \equiv \epsilon_2 - \epsilon_1 \ll \epsilon_1$ and $\Delta\lambda^z \equiv \lambda_2^z - \lambda_1^z \ll \lambda_1^z$. We also assume that phase damping is dominant $\lambda_i^z \gg \lambda_i^+$ and $\lambda_i^z \gg \lambda_i^-$. In the limit when $\Delta\epsilon$, $\Delta\lambda^z$, λ_i^+, and λ_i^- vanish, Eq. (0.6) reduces to Eq. (0.2). Then let us consider the following. In a short period of time T_0/N, under the Hamiltonian $\mathbf{H}_T$, the encoded state evolves into

$$\begin{aligned} &|\Psi(T_0/N)\rangle \\ &\approx [1 - i\mathbf{H}(T_0/N)](\alpha|01\rangle + \beta|10\rangle) \otimes |\Psi_b(0)\rangle \\ &= (\alpha|01\rangle + \beta|10\rangle) \otimes \\ &\quad [1 - 0i(T_0/N)(\epsilon_1 + \lambda_1^z V_z) - i(T_0/N)\mathbf{H}_B]|\Psi_b(0)\rangle \\ &\quad +(T_0/N)(-\alpha|01\rangle + \beta|10\rangle) \otimes (\Delta\epsilon + \Delta\lambda^z V_z)|\Psi_b(0)\rangle \\ &\quad +(T_0/N)|00\rangle \otimes (\lambda_1^+\beta + \lambda_2^+\alpha)V_+|\Psi_b(0)\rangle \\ &\quad +(T_0/N)|11\rangle \otimes (\lambda_1^-\alpha + \lambda_2^-\beta)V_-|\Psi_b(0)\rangle], \end{aligned} \tag{0.7}$$

where $|\Psi_b(0)\rangle$ denotes the bath state. Then we perform a measurement that discriminates between the encoding space Span$\{|01\rangle, |10\rangle\}$ and Span $\{|00\rangle, |11\rangle\}$. This measurement can be implemented by XORing each qubit to an ancilla qubit consecutively [14]. By frequently (i.e., N is made large) repeating time evolution by the Hamiltonian in Eq. (0.7) and the consecutive measurements, we can make effects of the terms involving $|00\rangle$ and $|11\rangle$ in Eq. (0.7) negligible (QZE). Then, after some simple calculations, we obtain

$$\begin{aligned} |\Psi(T_0)\rangle &\approx (\alpha|01\rangle + \beta|10\rangle) \otimes |\Psi_b\rangle \\ &\quad +O(T_0)(-\alpha|01\rangle + \beta|10\rangle) \otimes |\Psi_b'\rangle, \end{aligned} \tag{0.8}$$

where $|\Psi_b\rangle$ and $|\Psi_b'\rangle$ are some arbitrary bath states which are not necessarily orthogonal to each other. We can see that overall time evolution generates only phase errors in the encoded bit. In other words, the QZE prevents leakage of the states out of the encoding space while does not prevent time evolution within the encoding space. However, the QZE can be practicable for only fairly stable quantum states. Typical systems are well described by the model assumed here and thus the encoded qubit is fairly stable. Therefore, in this case the QZE can be a suitable choice for protecting the encoded qubit. When $\Delta\epsilon$ and $\Delta\lambda^z V_z$ are negligible, the second term of the right hand side of Eq. (0.8) is negligible and thus we need no more MRD. The two-qubit DFS in Eq. (0.5) plus the QZE is sufficient for preservation of one qubit. When they are not, we should reduce the effect of the term. This can be done in two ways, as noted in the Introduction. First, we concatenate the DFS in Eq. (0.5) with a three-qubit QECC that corrects only the phase errors (Eq. (15) of [11]). In this case, six qubits are needed to encode one qubit in the proposed scheme. Secondly, we concatenate the DFS in Eq.(0.5) with a two-qubit quantum code that detects only phase errors. That is,

$$\begin{aligned} |0_{enc}\rangle &= (|0\rangle + |1\rangle)(|0\rangle + |1\rangle), \\ |1_{enc}\rangle &= (|0\rangle - |1\rangle)(|0\rangle - |1\rangle), \end{aligned} \tag{0.9}$$

where $|0\rangle$ and $|1\rangle$ denote encoded qubits using the two-qubit DFS in Eq. (0.5) and normalization factors are omitted. Then we preserve the states using the QZE again: We frequently perform measurements that tell us whether the error has occurred or not [22]. In this case, four qubits are needed to encode one qubit in the proposed scheme. The first proposed scheme (two-qubit DFS + QZE) is efficient when replica asymmetry is negligible ($\Delta\lambda_z \approx 0$ and $\Delta\epsilon \approx 0$) and other terms (λ_i^+ and λ_i^-) are small but non-negligible. The second ([two-qubit DFS + QZE]× three-qubit QECC) and third ([two-qubit DFS + QZE]× [two-qubit quantum error detecting code + QZE]) proposed schemes are efficient when replica asymmetry is also non-negligible.

In Duan and Guo's scheme [23], the subspace that is orthogonal to the space to which the subspaces leak through the interaction Hamiltonian is adopted as the encoding space. Then the QZE is used for preventing leakage of qubits out of the encoding space. In contrast, in our scheme the qubit is first stabilized using DFS and then leakage is prevented by the QZE and time evolution within the encoding space is corrected by other MRDs (QECC or quantum error detecting code plus QZE). So the encoding space of Duan and Guo's scheme differs from that of our scheme for a given Hamiltonian. For example, in the case of the model of Eq. (0.2), the encoding space of Duan and Guo's scheme is Span$[|\bar{0}\bar{1}\rangle, |\bar{1}\bar{0}\rangle]$ where $|\bar{0}\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ and $|\bar{1}\rangle = (1/\sqrt{2})(|0\rangle - |1\rangle)$. This differs from the encoding space Span$[|01\rangle, |10\rangle]$ of our scheme. Duan and Guo's scheme [23] is more powerful than ours in that theirs is effective for wide classes of decoherence, i.e., for independent and even cooperative decoherence. In contrast, our scheme is a specialized one that is efficient in the case where phase errors are dominant but other errors are still non-negligible.

Here we proposed three schemes that are efficient when

each pair of qubits undergoes some imperfect collective decoherence with different baths. In the first scheme, each pair of qubits is encoded in a DFS composed of two qubits. Leakage out of the encoding space generated by the imperfection is reduced by the quantum Zeno effect. In the second scheme, phase errors in the encoded bits also generated by imperfection of replica symmetry are reduced by concatenation of the DFS with a three-qubit QECC that corrects only phase errors. In the third scheme, the same thing is done by concatenation of the DFS with two-qubit quantum error detecting code that detects only phase errors plus the QZE again.

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